$U_e(1)$–covariant R_ξ–gauge for the two Higgs doublet model

C. G. Honorato[(a)] and J. J. Toscano[(a,b)]

[(a)]*Facultad de Ciencias Físico Matemáticas, Benemérita Universidad*
Autónoma de Puebla, Apartado Postal 1152, Puebla, Puebla, México.
[(b)]*Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo,*
Edificio C-3, Ciudad Universitaria, C.P. 58040, Morelia, Michoacán, México.

An $U_e(1)$–covariant R_ξ gauge for the two Higgs doublet model based in the BRST symmetry is introduced. This gauge allows one to remove an important number of nonphysical vertices appearing in conventional linear gauges, which greatly simplifies the loop calculations, since the the resultant theory satisfies QED–like Ward identities. The presence of four ghost interactions in this type of gauges and its connection with the BRST symmetry is stressed. The Feynman rules for those new vertices that arise in this gauge, as well as for those couplings already present in the linear R_ξ gauge but that are modified by this gauge–fixing procedure, are presented.

I. INTRODUCTION

While gauge invariance plays a central role when defining the classical action of a gauge system, once the latter is quantized one must invariably invoke an appropriate gauge–fixing procedure to define a nondegenerate action, which means that gauge invariance is to broken explicitly [1]. The resultant action is not gauge invariant, though it is invariant under BRST symmetry [2]. As a consequence, the Green functions derived from this action cannot satisfy simple (QED–like) Ward identities, but they do satisfy more elaborated Slavnov–Taylor identities that are dictated by BRST symmetry. Although in conventional quantization schemes the quantum action of the theory is not gauge–invariant, it is still possible to introduce gauge invariance with respect to a subgroup of such a theory. The simplest example of this quantization scheme is the nonlinear gauge introduced by K. Fujikawa [3] in the context of the electroweak theory, which allows one to construct a quantum action that is invariant under the electromagnetic gauge group. This gauge–fixing procedure has been widely used in radiative corrections, as it greatly simplifies the loop calculations [4]. More recently, this quantization scheme has been generalized to include larger gauge groups than the electroweak one. In Ref. [5], one gauge–fixing procedure covariant under the $SU_L(2) \times U_Y(1)$ group was defined to quantize an electroweak extension based in the $SU_L(3) \times U_X(1)$ gauge group [6].

The main goal of this work is to introduce a general renormalizable nonlinear gauge-fixing procedure for the two Higgs doublet model (THDM), which is intended to remove the most nonphysical vertices from the interaction Lagrangian, thereby facilitating the calculation of radiative corrections considerably. The relevance of nonlinear R_ξ gauges [10] in radiative corrections has been emphasized by several authors not only within the context of the standard model (SM) [7, 10], but in some of its extensions such as the THDM [11], the so-called 331 model [5], and also in the model independent effective Lagrangian approach [12]. In this paper we will discuss to some extent the most general structure of a nonlinear R_ξ gauge for the THDM. We will argue that the Faddeev-Popov method (FPM) [13] fails when applied to this class of gauges because the resultant theory is nonrenormalizable. Instead, the appropriate framework is BRST symmetry [2], which is a powerful formalism suited to quantize Yang-Mills theories with broader supplementary conditions, and also more general gauge systems. To make clear this point, let us remind that the FPM leads to an action which is bilinear in the ghost and antighost fields as they arise essentially from the integral representation of a determinant [13]. However, this is not the most general situation that can arise since an action including four-ghost interactions at the tree level is still consistent with BRST symmetry and the power counting criterion of renormalization theory. It turns out that the FPM succeeds when used with conventional linear gauges [1] because no four-ghost interaction can arise from loop effects due to the existence of antighost translation invariance, [1] which in turn arises as a consequence of the fact that the antighost fields appear just through their derivatives. However, in the case of a nonlinear gauge, antighost translation invariance is lost as the gauge-fixing functions depend on bilinear terms of the gauge fields. These terms are responsible for the appearance of ultraviolet-divergent four-ghosts interactions at the one-loop level. This means that renormalizability becomes ruined when the FPM is used in conjunction with nonlinear gauges. It is thus convenient to desist from the use of the FPM and construct instead the most general action consistent with BRST symmetry and renormalization theory. Although these facts have been

---

[1] Invariance under the transformation $\bar{C}^a \to \bar{C}^a + c^a$, with c^a an arbitrary constant parameter

long known from the study of Yang-Mills theories without spontaneous symmetry breaking (SSB) [14], they are less known in the context of the SM and its extensions. Only more recently a complete Lagrangian for the ghost sector of the SM was presented [15]. In this work we pursue this issue and present a comprehensive study of nonlinear R_ξ gauges within the context of the THDM. Apart of eliminating much of the nonphysical vertices appearing in the THDM, our gauge–fixing procedure reduces in an appropriate limit to the corresponding nonlinear procedure for the SM given in Ref.[15].

The paper is organized as follows. In Sec. II, the main features of the THDM are discussed. Sec. III is devoted to present the most general nonlinear R_ξ gauge for the THDM. Finally in Sec. IV the conclusions are presented.

II. THE MODEL

We turn now to the main features of the Higgs sector of the THDM. We will focus on the CP-conserving Higgs potential, which is necessary to contextualize the nonlinear R_ξ gauge. We will also emphasize the phenomenological importance of the model and introduce the notation and conventions used throughout the rest of the paper. The THDM incorporates two scalar doublets of hypercharge +1: $\Phi_1^\dagger = (\phi_1^-, \phi_1^{0*})$ and $\Phi_2^\dagger = (\phi_2^-, \phi_2^{0*})$. The most general gauge invariant potential can written as

$$
\begin{aligned}
V(\Phi_1, \Phi_2) \;=\; & \mu_1^2(\Phi_1^\dagger\Phi_1) + \mu_2^2(\Phi_2^\dagger\Phi_2) - \left(\mu_{12}^2(\Phi_1^\dagger\Phi_2) + \text{H.c.}\right) + \lambda_1(\Phi_1^\dagger\Phi_1)^2 \\
& +\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
& +\frac{1}{2}\left(\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left(\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right)(\Phi_1^\dagger\Phi_2) + \text{H.c.}\right)
\end{aligned}
\tag{1}
$$

It is usual to impose the discrete symmetry $\Phi_1 \to \Phi_1$ and $\Phi_2 \to -\Phi_2$ in order to avoid dangerous flavor changing neutral current (FCNC) effects. This symmetry is strongly violated by the λ_6 and λ_7 terms, but it is softly violated by μ_{12}^2. Nevertheless, all of these terms are essential to obtain the decoupling limit of the model in which only one CP-even scalar is light. As long as these violating terms exist, there are two independent energy scales [16], v (the Fermi scale) and Λ_{THDM}, and the spectrum of Higgs boson masses is such that m_h is of the order of v, whereas m_H, m_A and $m_{H^\pm}$ are all of the order of Λ_{THDM}. In this case, all of the heavy Higgs bosons decouple in the limit of $\Lambda_{\text{THDM}} \gg v$, according to the decoupling theorem. On the other hand, when the scalar potential does respect the discrete symmetry, it is impossible to have two independent energy scales [16]. As a consequence, all of the physical scalar masses lie on the Fermi scale v. Since v is already fixed by the experiment, a very heavy Higgs boson can only arise through a large dimensionless coupling constant λ_i. In this scenario the decoupling theorem is no longer valid, thereby opening the possibility for the appearance of nondecoupling effects. In addition, since the scalar potential contains some terms that violate the $SU(2)$ custodial symmetry, nondecoupling effects can arise in one-loop induced Higgs boson couplings [17].

The scalar potential (1) has to be diagonalized to yield the mass-eigenstates fields. The charged components of the doublets lead to a physical charged Higgs boson and the pseudo-Goldstone boson associated with the W gauge field:

$$
G_W^\pm = \phi_1^\pm c_\beta + \phi_2^\pm s_\beta,
\tag{2}
$$

$$
H^\pm = -\phi_1^\pm s_\beta + \phi_2^\pm c_\beta,
\tag{3}
$$

with $\tan\beta = v_2/v_1$, being $v_1/\sqrt{2}$ $(v_2/\sqrt{2})$ the vacuum expectation value (VEV) associated with Φ_1 (Φ_2), and $m_{H^\pm}^2 = -(v_1^2 + v_2^2)(\lambda_4 + \lambda_5 - 2\mu_{12}^2/(v_1 v_2))/2$. Also, we have included the shorthand notation $s_\beta = \sin\beta$ and $c_\beta = \cos\beta$. On the other hand, the imaginary part of the neutral components ϕ_{iI}^0 defines the neutral CP-odd scalar and the pseudo-Goldstone boson associated with the Z gauge boson. The corresponding rotation is given by

$$
G_Z = \phi_{1I}^0 c_\beta + \phi_{2I}^0 s_\beta,
\tag{4}
$$

$$
A = -\phi_{1I}^0 s_\beta + \phi_{2I}^0 c_\beta,
\tag{5}
$$

where $m_A^2 = -(v_1^2 + v_2^2)(\lambda_5 - \mu_{12}^2/(v_1 v_2))$. Finally, the real part of the neutral components of the ϕ_{iR}^0 doublets defines the CP-even Higgs bosons h and H. The mass matrix is given by

$$
M_{Re} = \begin{pmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{pmatrix},
\tag{6}
$$

where

$$m_{11} = 2v_1^2\lambda_1 + \frac{v_2}{v_1}\mu_{12}^2, \tag{7}$$

$$m_{22} = 2v_2^2\lambda_2 + \frac{v_2}{v_1}\mu_{12}^2, \tag{8}$$

$$m_{12} = v_1v_2(\lambda_3 + \lambda_4 + \lambda_5) - \mu_{12}^2. \tag{9}$$

The physical CP-even states, h and H, are given by

$$H = \phi_{1R}^0 c_\alpha + \phi_{2R}^0 s_\alpha, \tag{10}$$
$$h = -\phi_{1R}^0 s_\alpha + \phi_{2R}^0 c_\alpha, \tag{11}$$

where

$$\tan 2\alpha = \frac{2m_{12}}{m_{11} - m_{22}}, \tag{12}$$

and

$$m_{H,h}^2 = \frac{1}{2}\left(m_{11} + m_{22} \pm \sqrt{(m_{11} - m_{22})^2 + 4m_{12}^2}\right). \tag{13}$$

III.   THE GAUGE-FIXING PROCEDURE

As already mentioned, the FPM fails to quantize Yang-Mills theories with more general supplementary conditions than the linear ones. To quantize Yang-Mills theories involving more general gauge-fixing functions or either more general gauge systems, it is thus necessary to invoke BRST symmetry. Although this symmetry can be implemented in conventional field theory [2, 18], it arises more naturally within the context of the field-antifield formalism [19]. Therefore, in order to clarify our presentation as much as possible, we will present a brief discussion of this formalism. Although the following discussion is rather general, we will focus on the properties of Yang-Mills systems. The starting point is the introduction of an antifield for each field present in the theory. It is assumed that the dynamical degrees of freedom of the gauge system are characterized by the matter, gauge, ghost (C^a), antighost ($\bar{C}^a$), and auxiliary (B^a) fields. The original action, which will be denoted by S_0, is a functional of the matter and the gauge fields only, but this configuration is extended to include the ghost fields because they are necessary to quantize the theory. A ghost field for each gauge parameter is introduced. The ghost fields have opposite statistics to that of the gauge parameters. To gauge fix and quantize the theory, it is necessary to introduce the so-called trivial pairs, namely the antighost and auxiliary fields. We let Φ^A run over all these fields. For each Φ^A, an antifield Φ_A^* is introduced, with opposite statistics to Φ^A and a ghost number equal to $-gh(\Phi^A) - 1$, where $gh(\Phi^A)$ is the ghost number of Φ^A. It is 0 for matter, gauge, and auxiliary fields, $+1$ for ghosts, and -1 for antighosts. In this extended configuration space a symplectic structure is introduced through left and right differentiation, defined for two functionals F and G as:

$$(F, G) = \frac{\delta_R F}{\delta \Phi^A}\frac{\delta_L G}{\delta \Phi_A^*} - \frac{\delta_R F}{\delta \Phi_A^*}\frac{\delta_L G}{\delta \Phi^A}. \tag{14}$$

In particular, the fundamental antibrackets are given by

$$(\Phi^A, \Phi_B^*) = \delta_B^A, \tag{15}$$
$$(\Phi^A, \Phi^B) = (\Phi_A^*, \Phi_B^*) = 0. \tag{16}$$

The extended action is a bosonic functional of the fields and the antifields, $S[\Phi, \Phi^*]$, with ghost number zero, which satisfies the master equation defined by

$$(S, S) = 2\frac{\delta_R S}{\delta \Phi^A}\frac{\delta_L S}{\delta \Phi_A^*} = 0. \tag{17}$$

The antibrackets serve to define the extended BRST transformations:

$$\delta_B \Phi^A = (S, \Phi^A) = -\frac{\delta_R S}{\delta \Phi_A^*}, \tag{18}$$

$$\delta_B \Phi_A^* = (S, \Phi_A^*) = \frac{\delta_R S}{\delta \Phi^A}. \tag{19}$$

Thus, the BRST transformations are generated by the extended action, which is invariant under these transformations due to the master equation as its variation is $\delta_B S = (S, S)$. On the other hand, not all the solutions of the master equation are of interest, but only those called proper solutions [19]. A proper solution must make contact with the initial theory, which means to impose the following boundary condition on S:

$$S[\Phi, \Phi^*]|_{\Phi^*=0} = S_0[\phi], \tag{20}$$

where ϕ runs only over the original fields, i.e., matter and gauge fields. The proper solution S can be expanded in a power series in antifields:

$$S[\Phi, \Phi^*] = S_0[\phi] + (\delta_B \Phi^A)\Phi_A^* + \cdots, \tag{21}$$

in which all the gauge-structure tensors characterizing the gauge system appear. In this sense, the proper solution S is the generating functional of the gauge-structure tensors. S also generates the gauge algebra through the master equation. So, classically a gauge system is completely determined when the proper solution S is established and the master equation is calculated, which yields the relations that must be satisfied by the gauge-structure tensors. In the simplest gauge systems, such as Yang-Mills theories, a solution of the master equation is given by

$$S[\Phi, \Phi^*] = S_0[\phi] + (\delta_B \Phi^A)\Phi_A^*. \tag{22}$$

This action is bosonic and has ghost number zero as required. It is easy to show that this action is a solution of the master equation and accurately reproduces the well-known gauge algebra of Yang-Mills theories.

We turn now to the quantum analysis of the gauge system. To quantize the theory, one starts by fixing the gauge. Since the extended action is degenerate, it cannot be quantized directly. Furthermore, the antifields do not represent true degrees of freedom, so they must be removed before quantizing the theory. However they cannot be just set to zero since S_0 is degenerate. One can remove the antifields instead through a nontrivial procedure and at the same time lift the degeneration of the theory. The antifields can be eliminated by introducing a fermionic functional of the fields, $\Psi[\Phi]$, with ghost number -1, such that

$$\Phi_A^* = \frac{\delta \Psi[\Phi]}{\delta \Phi^A}. \tag{23}$$

Note that it is not necessary to distinguish between left- and right-differentiation. In defining a gauge-fixing procedure, the presence of the trivial pairs, $\bar{C}^a$ and B^a, is necessary since the only fields with ghost number -1 are precisely the antighosts. By noting that $(\delta_B \Phi^A)\Phi_A^* = (\delta_B \Phi^A)(\delta \Psi[\Phi]/\delta \Phi^A) = \delta \Psi[\Phi]$, the proper solution takes the form

$$S[\Phi, \delta\Psi/\delta\Phi] = S_0[\phi] + \delta\Psi[\Phi]. \tag{24}$$

This is the gauge-fixed BRST action, which is invariant under the well-known BRST transformations [2]. From now on, we will denote these transformations by the symbol δ. It should be noted that, in contrast with the extended action S, $S_0 + \delta\Psi[\Phi]$ is not degenerate. In general, the nilpotency of δ is only guaranteed on-shell, i.e., only after using the equations of motion, but in the case of Yang-Mills theories, $\delta^2 = 0$ even off-shell.

We now proceed to define the most general fermionic functional $\Psi[\Phi]$ for the electroweak group $SU_L(2) \times U_Y(1)$, consistent with renormalization theory. The most general Ψ functional with ghost number -1 can be written as follows:

$$\Psi = \int d^4x \Big[\bar{C}^a \Big(f^a + \frac{\xi}{2} B^a + g\epsilon^{abc} \bar{C}^b C^c \Big) + \bar{C}\Big(f + \frac{\xi}{2} B \Big) \Big], \tag{25}$$

where f^a and f are the gauge-fixing functions associated with the groups $SU_L(2)$ and $U_Y(1)$, respectively. They are restricted by renormalizability to be at most quadratic functions of the gauge and the scalar fields. The bosonic constant ξ is the so-called gauge parameter. In general there is one such parameter for each group, but we have used the same by simplicity. B^a and B are the auxiliary fields associated with the groups $SU_L(2)$ and $U_Y(1)$, respectively. Note that the term $g\epsilon^{abc}\bar{C}^a\bar{C}^b C^c$ cannot exist in the Faddeev-Popov approach, though its presence is necessary to get

renormalizability when the gauge-fixing functions are nonlinear. Using the usual BRST transformations, we obtain for the Ψ variation

$$
\begin{aligned}
\delta\Psi &= \int d^4x \Big\{ \frac{\xi}{2} B^a B^a + g\Big(f^a + 2\epsilon^{abc}\bar{C}^b C^c \Big) B^a + \frac{\xi}{2} BB + fB \\
&\quad - \bar{C}^a(\delta_{B_\Psi} f^a) - \bar{C}(\delta_{B_\Psi} f) - g^2 \bar{C}^a \bar{C}^b C^a C^b \Big\}.
\end{aligned}
\tag{26}
$$

On the other hand, since the auxiliary fields B^a and B appear quadratically, they can be integrated out. Since the coefficients of the quadratic terms do not depend on the fields, their integration is equivalent to use the corresponding equations of motion in the gauge-fixed BRST action. Once this is done, we obtain an effective action defined by the following effective Lagrangian

$$
\mathcal{L}_{eff} = \mathcal{L}_{THDM} + \mathcal{L}_B + \mathcal{L}_F,
\tag{27}
$$

where $\mathcal{L}_{THDM}$ is the gauge invariant Lagrangian of the THDM, whereas $\mathcal{L}_B$ and $\mathcal{L}_F$ arise from the action $\delta\Psi$. $\mathcal{L}_B$ is the gauge-fixing term, which can be written as

$$
\mathcal{L}_B = -\frac{1}{2\xi} f^a f^a - \frac{1}{2\xi}(f)^2.
\tag{28}
$$

On the other hand, $\mathcal{L}_F$ represents the ghost sector and it is given by

$$
\begin{aligned}
\mathcal{L}_F &= -\bar{C}^a(\delta f^a) - \bar{C}(\delta f) \\
&\quad - \frac{2g}{\xi}\epsilon^{abc} f^a \bar{C}^b C^c + g^2\Big(\frac{2}{\xi} - 1\Big)\bar{C}^a \bar{C}^b C^a C^b.
\end{aligned}
\tag{29}
$$

While the first two terms correspond to those arising from the FPM, the last two ones do not appear when this framework is used. It is important to note that the $2g/\xi\,\epsilon^{abc} f^a \bar{C}^b C^c$ term can contribute to some class of Green functions yet at the one-loop level, a fact to be considered with dealing with radiative corrections.

A.   The nonlinear gauge-fixing functions

We are now ready to discuss the gauge-fixing functions f^a and f in the context of the THDM. Our main aim is to remove the most unphysical vertices that are generated by the Higgs kinetic-energy term. We will take advantage of the fact that every coupling involving at least one pseudo-Goldstone boson can be modified or removed from the theory leaving unaltered the S matrix. Also, gauge freedom allows us to modify the Yang-Mills sector in a nontrivial way. Bearing this in mind, the first step in our strategy consist in defining a gauge-fixing procedure for the charged W gauge field covariant under the $U_e(1)$ group. This means that the covariant derivative associated with this group must be used instead of the ordinary one. An adequate extension of this derivative allows us to eliminate not only the unphysical vertex $WG_W\gamma$ but also the $WG_W Z$ one, and simultaneously guarantees a highly symmetric behavior of the theory. The second stage in our program consists in introducing gauge-fixing functions nonlinear in the scalar sector, meant to remove various unphysical quartic vertices, which involve interactions between gauge bosons, pseudo-Goldstone bosons, and physical scalar bosons. We thus propose the following gauge-fixing functions for the THDM:

$$
f^a = f_V^a + f_S^a,
\tag{30}
$$

$$
f = f_V + f_S,
\tag{31}
$$

where

$$
f_V^a = \Big(\delta^{ab}\partial_\mu - g'\epsilon^{3ab} B_\mu \Big) W^{b\mu},
\tag{32}
$$

$$
\begin{aligned}
f_S^a &= \frac{ig\xi}{2}\Big\{ \sum_{i=1}^{2} \Big[\Phi_i^\dagger(\sigma^a - i\epsilon^{3ab}\sigma^b)\Phi_{0i} - \Phi_{0i}^\dagger(\sigma^a + i\epsilon^{3ab}\sigma^b)\Phi_i \Big] \\
&\quad + i\epsilon^{3ab}(c_\beta\Phi_1^\dagger + s_\beta\Phi_2^\dagger)\sigma^b(c_\beta\Phi_1 + s_\beta\Phi_2) \Big\},
\end{aligned}
\tag{33}
$$

and

$$f_V = \partial_\mu B^\mu,\tag{34}$$

$$f_S = \frac{ig'\xi}{2}\sum_{i=1}^{2}\left(\Phi_i^\dagger\Phi_{0i} - \Phi_{0i}^\dagger\Phi_i\right).\tag{35}$$

In the above expressions, $\Phi_{0i}^\dagger = (0, v_i/\sqrt{2})$, σ^a are the Pauli matrices, and W_μ^a and B_μ are the gauge fields associated with the electroweak group. Our gauge-fixing functions contain the conventional linear functions as a particular case. In fact, they are obtained when ϵ^{3ab} is set to zero. All these functions are Hermitian as required. Also, we have introduced the linear combination of Higgs doublets $c_\beta\Phi_1 + s_\beta\Phi_2$, which is necessary in order to avoid the presence of terms involving only physical fields. In other words, any term appearing in f_S^a and f_S involves the presence of at least one unphysical scalar. It is worth mentioning that this gauge-fixing procedure contains as a particular case an analogous gauge scheme for the minimal SM [15], which becomes evident when the Φ_1 doublet is associated with the SM one and β is set to zero

To fully appreciate the structure of the gauge-fixing functions, it is convenient to express them in terms of mass eigenstates fields. To this end, we use the following definitions:

$$f^\pm = \frac{1}{\sqrt{2}}(f^1 \mp if^2),\tag{36}$$

$$f^Z = c_W f^3 - s_W f,\tag{37}$$

$$f^A = s_W f^3 + c_W f,\tag{38}$$

where s_W (c_W) is the sine (cosine) of the weak angle. We then obtain for the vector sector

$$f_V^+ = \bar{D}_\mu W^{+\mu},\tag{39}$$

$$f_V^Z = \partial_\mu Z^\mu,\tag{40}$$

$$f_V^A = \partial_\mu A^\mu,\tag{41}$$

and for the scalar sector

$$f_S^+ = -\frac{ig\xi}{2}\left(\varphi^0 - iG_Z\right)G_W^+,\tag{42}$$

$$f_S^Z = -\xi m_Z G_Z,\tag{43}$$

$$f_S^A = 0,\tag{44}$$

where $\varphi^0 = v + c_{\beta-\alpha}H + s_{\beta-\alpha}h$ and $\bar{D}_\mu = \partial_\mu - ig'B_\mu$, being g' the coupling constant associated with the $U_Y(1)$ group. We can see that both f_V^+ and f_S^+ are nonlinear and transform covariantly under the $U_e(1)$ group, as $\bar{D}_\mu$ contains the covariant derivative associated with this group.

The gauge-fixing Lagrangian, $\mathcal{L}_B$, can then be written as

$$\mathcal{L}_B = \mathcal{L}_{BV} + \mathcal{L}_{BS} + \mathcal{L}_{BSV},\tag{45}$$

where

$$\begin{aligned}\mathcal{L}_{BV} &= -\frac{1}{\xi}f_V^- f_V^+ - \frac{1}{2\xi}(f_V^Z)^2 - \frac{1}{2\xi}(f_V^A)^2\\ &= -\frac{1}{\xi}(\bar{D}_\mu W^{+\mu})^\dagger(\bar{D}_\nu W^{+\nu}) - \frac{1}{2\xi}(\partial_\mu Z^\mu)^2 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2,\end{aligned}\tag{46}$$

$$\begin{aligned}\mathcal{L}_{BS} &= -\frac{1}{\xi}f_S^- f_S^+ - \frac{1}{2\xi}(f_S^Z)^2\\ &= -\frac{g^2\xi}{4}\left(\varphi^2 + G_Z^2\right)G_W^- G_W^+ - \frac{1}{2}\xi m_Z^2 G_Z^2,\end{aligned}\tag{47}$$

$$\begin{aligned}\mathcal{L}_{BSV} &= -\frac{1}{\xi}\left(f_V^- f_S^+ + f_S^- f_V^+\right) - \frac{1}{2\xi}f_V^Z f_S^Z\\ &= \frac{ig}{2}\left[(\bar{D}_\mu W^{+\mu})^\dagger(\varphi - iG_Z)G_W^+ - (\bar{D}_\mu W^{+\mu})(\varphi + iG_Z)G_W^-\right] + m_Z G_Z \partial_\mu Z^\mu.\end{aligned}\tag{48}$$

We now proceed to investigate the consequences of these gauge-fixing terms on the gauge invariant Lagrangian of the THDM. Feynman rules for the THDM in conventional linear gauges have been discussed by several authors [16, 20, 21, 22, 23, 24]. Here, we will present only those couplings which are affected by our nonlinear gauge–fixing procedure. First of all, the term $\mathcal{L}_{BV}$ defines the propagators of the gauge fields and also modifies nontrivially the Lorentz structure of the trilinear and quartic vertices arising from the Yang-Mills sector. To clarify this point, it is convenient to write down the sum of these two terms:

$$
\begin{aligned}
\mathcal{L}_V &= \mathcal{L}_{YM} + \mathcal{L}_{BV} \\
&= -\frac{1}{2}(\hat{D}_\mu W_\nu^+ - \hat{D}_\nu W_\mu^+)^\dagger(\hat{D}^\mu W^{+\nu} - \hat{D}^\nu W^{+\mu}) \\
&\quad -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
&\quad -ig\left[s_W F_{\mu\nu} + c_W Z_{\mu\nu} + \frac{ig}{2}(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+)\right]W^{-\mu}W^{+\nu} \\
&\quad -\frac{1}{\xi}(\bar{D}_\mu W^{+\mu})^\dagger(\bar{D}_\nu W^{+\nu}) - \frac{1}{2\xi}(\partial_\mu Z^\mu)^2 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2,
\end{aligned}
\tag{49}
$$

where $\hat{D}_\mu = \partial_\mu - igW_\mu^3$, being g the constant coupling associated with the $SU_L(2)$ group. It is thus evident that, with the exception of the $WWWW$ vertex, all trilinear and quartic vertices are modified by the gauge fixing procedure. Since the term that introduces the modifications on these vertices is invariant under the $U_e(1)$ group, the trilinear electromagnetic vertices satisfy QED-like Ward identities. This fact is relevant for radiative corrections as such a symmetry greatly simplifies this class of calculations. On the other hand, as it can be appreciated from the above Lagrangian, the trilinear WWV ($V = \gamma, Z$) vertex is modified in a nontrivial way by this gauge–fixing procedure. With the exception of the $WWWW$ vertex, the quartic interactions $WWVV$ are modified too. They now are gauge depending, i.e., they depend on the ξ gauge parameter. The vertex functions associated with the $V^\mu(k_1)W^{+\lambda}(k_2)W^{-\rho}(k_3)$ and $V_1^\mu V_2^\nu W^{+\lambda}W^{-\rho}$ couplings are given by $ig_V\Gamma_{\lambda\rho\mu}(k_1,k_2,k_3)$ and $ig_{V_1}g_{V_2}\Gamma_{\lambda\rho\mu\nu}$, respectively, where

$$
\begin{aligned}
\Gamma_{\lambda\rho\mu}(k_1,k_2,k_3) &= (k_2-k_3)_\mu g_{\lambda\rho} + (k_3-k_1+\frac{\delta_V}{\xi}k_2)_\lambda g_{\rho\mu} \\
&\quad +(k_1-k_2-\frac{\delta_V}{\xi}k_3)_\rho g_{\lambda\mu},
\end{aligned}
\tag{50}
$$

$$
\Gamma_{\lambda\rho\mu\nu} = -2g_{\mu\nu}g_{\lambda\rho} + \left(1 - \frac{\delta_{V_1V_2}}{\xi}\right)(g_{\lambda\mu}g_{\rho\nu} + g_{\rho\mu}g_{\lambda\nu}),
\tag{51}
$$

where all momenta are token incoming. In addition, we have introduced the following definitions:

$$
\delta_{V_1V_2} = \begin{cases} +1, & V_1V_2 = \gamma\gamma \\ -\frac{s_W^2}{c_W^2}, & V_1V_2 = \gamma Z, \\ +\frac{s_W^4}{c_W^4}, & V_1V_2 = ZZ, \end{cases}
\tag{52}
$$

$$
g_V = \begin{cases} gs_W, & V = \gamma \\ gc_W, & V = Z. \end{cases}
\tag{53}
$$

As for the $\mathcal{L}_{BS}$ term, it defines the masses of the G_W and G_Z fields and modifies some unphysical couplings arising from the Higgs potential. In this gauge, the couplings between scalar fields arise solely from the sum of the following terms

$$
\begin{aligned}
\mathcal{L}_S &= -V(\Phi_1,\Phi_2) + \mathcal{L}_{BS} \\
&= -V(\Phi_1,\Phi_2) - \frac{1}{2}\xi m_Z^2 G_Z^2 - \frac{g^2\xi}{4}(\varphi^{02} + G_Z^2)G_W^- G_W^+.
\end{aligned}
\tag{54}
$$

From this expression, it is clear that the gauge-fixing procedure modifies the strength of the couplings $HG_W^- G_W^+$, $hG_W^- G_W^+$, $H^2 G_W^- G_W^+$, $h^2 G_W^- G_W^+$, $Hh G_W^- G_W^+$, and $G_Z^2 G_W^- G_W^+$. The modified Feynman rules are shown in Table I. The physical couplings remain unchanged, as required.

TABLE I: Nonphysical couplings of the Higgs potential that are modified by the nonlinear gauge–fixing procedure.

Coupling	Vertex Function
$HHG_W^+G_W^-$	$\frac{-ig^2}{4m_W^2}\left(2m_{H^\pm}^2 s_{\alpha+\beta}^2 - \frac{m_h^2 s_{2\alpha} s_{\alpha-\beta}^2}{s_{2\beta}} + \frac{m_H^2(s_{\beta-3\alpha}+3s_{\alpha+\beta})c_{\alpha-\beta}}{2s_{2\beta}} + 2m_W^2\xi c_{\alpha-\beta}^2\right)$
$hhG_W^+G_W^-$	$\frac{-ig^2}{4m_W^2}\left(2m_{H^\pm}^2 c_{\alpha-\beta}^2 + \frac{m_H^2 s_{2\alpha} c_{\alpha-\beta}^2}{s_{2\beta}} - \frac{m_h^2(c_{\beta-3\alpha}+3c_{\alpha+\beta})s_{\alpha-\beta}}{2s_{2\beta}} + 2m_W^2\xi s_{\alpha-\beta}^2\right)$
$G_Z G_Z G_W^+ G_W^-$	$\frac{-ig^2}{4m_W^2}\left(m_H^2 c_{\alpha-\beta}^2 + m_h^2 s_{\alpha-\beta}^2 + 2m_W^2\xi\right)$
$HhG_W^+G_W^-$	$\frac{-ig^2}{4m_W^2}\left(m_{H^\pm}^2 s_{2(\alpha-\beta)} - \frac{(m_h^2-m_H^2)s_{2\alpha}s_{2(\alpha-\beta)}}{2s_{2\beta}} - m_W^2\xi s_{2(\alpha-\beta)}\right)$
$HG_W^+G_W^-$	$\frac{-ig}{2m_W}\left(m_H^2 + 2m_W\xi\right)c_{\beta-\alpha}$
$hG_W^+G_W^-$	$\frac{-ig}{2m_W}\left(m_h^2 + 2m_W\xi\right)s_{\beta-\alpha}$

We now turn to discuss the dynamical implications of the term $\mathcal{L}_{BSV}$. This term affects considerably the Higgs kinetic-energy sector of the theory since it removes several unphysical vertices. When these two terms are combined, we obtain

$$\sum_{i=1}^{2}(D_\mu\Phi_i)^\dagger(D^\mu\Phi_i) + \mathcal{L}_{BSV} = \frac{1}{2}\sum_{\phi=h,H,A,G_Z}(\partial_\mu\phi)(\partial^\mu\phi) + \sum_{\phi^+=H^+,G_W^+}(\partial_\mu\phi^-)(\partial^\mu\phi^+)$$

$$+\frac{ig}{2}\{W_\mu^-[-2G_W^+\partial^\mu S_1 + 2iG_W^+\partial^\mu G_Z + S_2\partial^\mu H^+ - H^+\partial^\mu S_2$$

$$+i(H^+\partial^\mu A - A\partial^\mu H^+) - ie(A^\mu - \frac{s_W}{c_W}Z^\mu)(S_2 - iA)H^+] - h.c.\}$$

$$+\frac{g}{2c_W}Z_\mu[c_{\beta-\alpha}(S_2\partial^\mu P_1 - P_1\partial^\mu S_2 + S_1\partial^\mu P_2 - P_2\partial^\mu S_1)$$

$$+s_{\beta-\alpha}(S_1\partial^\mu P_1 - P_1\partial^\mu S_1 + P_2\partial^\mu S_2 - S_2\partial^\mu P_2)]$$

$$+ie(A_\mu + \frac{c_{2W}}{s_{2W}}Z_\mu)(G_W^-\partial^\mu G_W^+ - G_W^+\partial^\mu G_W^- + H^-\partial^\mu H^+ - H^+\partial^\mu H^-)$$

$$+e^2(A_\mu A^\mu + 2\frac{c_{2W}}{s_{2W}}A_\mu Z^\mu + \frac{c_{2W}^2}{s_{2W}^2}Z_\mu Z^\mu + \frac{1}{2s_W^2}W_\mu^- W^{+\mu})(G_W^- G_W^+ + H^- H^+)$$

$$+\frac{g^2}{4}(W_\mu^- W^{+\mu} + \frac{1}{2c_W^2}Z_\mu Z^\mu)(v^2 + 2vS_1 + h^2 + H^2 + A^2 + G_Z^2), \tag{55}$$

where

$$S_1 = c_{\beta-\alpha}H + s_{\beta-\alpha}h, \tag{56}$$
$$S_2 = -s_{\beta-\alpha}H + c_{\beta-\alpha}h, \tag{57}$$

$$P_1 = c_{\beta-\alpha}A + s_{\beta-\alpha}G_Z, \tag{58}$$
$$P_2 = -s_{\beta-\alpha}A + c_{\beta-\alpha}G_Z. \tag{59}$$

In conventional linear gauges, only the mixing terms $W - G_W$ and $Z - G_Z$ are removed from the theory. In contrast, our gauge-fixing procedure also removes the unphysical vertices $WG_W\gamma$, $WG_W Z$, $HWG_W\gamma$, $hWG_W\gamma$, $G_Z WG_W\gamma$, $HWG_W Z$, $hWG_W Z$, and $G_Z WG_W Z$. In addition, the unphysical vertices HWG_W, hWG_W, and $G_Z WG_W$ are modified. The Feynman rules for the modified vertices are given in Table II. Once again, note that the couplings involving only physical scalars are not modified by the gauge-fixing procedure.

TABLE II: Nonphysical coupling of the Higgs–Kinetic energy term that are affected by the nonlinear gauge–fixing procedure. Notices that the vertices which vanish in this gauge but not in conventional ones are indicated. In this Table, ϕ_a stands for h o H.

Coupling	Vertex Function	Coupling	Vertex Function
$H(k_1)W_\mu^\pm(k_2)G_W^\mp(k_3)$	$\mp igk_{1\mu}c_{\beta-\alpha}$	$h(k_1)W_\mu^\pm(k_2)G_W^\mp(k_3)$	$\mp igk_{1\mu}s_{\beta-\alpha}$
$G_Z(k_1)W_\mu^\pm(k_2)G_W^\mp(k_3)$	$gk_{1\mu}$	$V_\nu W_\mu^\pm G_W^\mp$	0
$V_\mu W_\nu^\pm \phi_a G_W^\mp$	0	$V_\mu W_\nu^\pm G_Z G_W^\mp$	0

B. The ghost sector

Let us now discuss the structure of the ghost sector. It is convenient to introduce the following definitions for the ghost fields

$$C^\pm = \frac{1}{\sqrt{2}}(C^1 \mp iC^2), \tag{60}$$

$$C^Z = c_W C^3 - s_W C, \tag{61}$$

$$C^A = s_W C^3 + c_W C, \tag{62}$$

and similar expressions for the antighost fields. We can then write the corresponding Lagrangian as follows:

$$\mathcal{L}_F = \mathcal{L}_{FV1} + \mathcal{L}_{FS1} + \mathcal{L}_{FV2} + \mathcal{L}_{FS2} + \mathcal{L}_{F3}, \tag{63}$$

where

$$\mathcal{L}_{FV1} = -\bar{C}^-(\delta f_V^+) - \bar{C}^+(\delta f_V^-) - \bar{C}^Z(\delta f_V^Z) - \bar{C}^A(\delta f_V^A), \tag{64}$$

$$\mathcal{L}_{FS1} = -\bar{C}^-(\delta f_S^+) - \bar{C}^+(\delta f_S^-) - \bar{C}^Z(\delta f_S^Z), \tag{65}$$

$$\mathcal{L}_{FV2} = -\frac{2ig}{\xi}\Big[(f_V^-\bar{C}^+ - f_V^+\bar{C}^-)(c_W C^Z + s_W C^A)$$
$$+(c_W\bar{C}^Z + s_W\bar{C}^A)(f_V^+ C^- - f_V^- C^+) + (c_W f_V^Z + s_W f_V^A)(\bar{C}^- C^+ - \bar{C}^+ C^-)\Big], \tag{66}$$

$$\mathcal{L}_{FS2} = -\frac{2ig}{\xi}\Big[(f_S^-\bar{C}^+ - f_S^+\bar{C}^-)(c_W C^Z + s_W C^A)$$
$$+(c_W\bar{C}^Z + s_W\bar{C}^A)(f_S^+ C^- - f_S^- C^+) + c_W f_S^Z(\bar{C}^- C^+ - \bar{C}^+ C^-)\Big], \tag{67}$$

$$\mathcal{L}_{F3} = 2g^2\Big(1 - \frac{2}{\xi}\Big)\Big[\bar{C}^+\bar{C}^- C^+ C^-$$
$$+(c_W\bar{C}^Z + s_W\bar{C}^A)(\bar{C}^+ C^- + \bar{C}^- C^+)(c_W C^Z + s_W C^A)\Big]. \tag{68}$$

It is important to point out that the terms $\mathcal{L}_{FV2}$, $\mathcal{L}_{FS2}$, and $\mathcal{L}_{F3}$ are not present when the FPM is used. The variations of the charged gauge-fixing functions are given by

$$\delta f_V^+ = \bar{D}_\mu\hat{D}^\mu C^+ + ig(c_W C^Z + s_W C^A)(\bar{D}_\mu W^{+\mu}) + \frac{ig}{c_W}W^{+\mu}(\partial_\mu C^Z), \tag{69}$$

$$\delta f_S^+ = \frac{g^2\xi}{2}\Big\{\Big[G_W^- G_W^+ - \frac{1}{2}(\varphi^{02} + G_Z^2)\Big]C^+ - (\varphi^0 - iG_Z)G_W^+(c_W C^Z + s_W C^A)\Big\}, \tag{70}$$

where $\delta f_{V,S}^- = (\delta f_{V,S}^+)^\dagger$. These functions transform covariantly under the $U_e(1)$ group. As for the variations of the neutral functions, they are given by

$$\delta f_V^Z = \Box C^Z + igc_W\partial_\mu(W^{-\mu}C^+ - W^{+\mu}C^-), \tag{71}$$

$$\delta f_S^Z = \frac{g\xi}{2}m_Z(G_W^+ C^- + G_W^- C^+) - \frac{g\xi}{2c_W}m_Z\varphi^0 C^Z, \tag{72}$$

$$\delta f_V^A = \Box C^A + ie\partial_\mu(W^{-\mu}C^+ - W^{+\mu}C^-). \tag{73}$$

TABLE III: Trilinear couplings involving ghost fields.

Coupling	Vertex Function
$\bar{C}^{\pm}C^{\mp}H$	$igm_W\xi c_{\beta-\alpha}$
$\bar{C}^{\pm}C^{\mp}h$	$igm_W\xi s_{\beta-\alpha}$
$\bar{C}^{\pm}C^{\mp}G_Z$	$\pm 2gm_Zc_W$
$\bar{C}^ZC^{\pm}G_W^{\mp}$	$\frac{-igm_Z\xi}{2}$
$\bar{C}^ZC^ZH$	$\frac{igm_Z\xi}{2c_W}c_{\beta-\alpha}$
$\bar{C}^ZC^Zh$	$\frac{igm_Z\xi}{2c_W}s_{\beta-\alpha}$
$\bar{C}^{\pm}(\bar{P})C^{\mp}(P)A_\mu(k)$	$\pm igs_W\left[\left(1+\frac{2}{\xi}\right)P - \left(1-\frac{2}{\xi}\right)\bar{P}\right]_\mu$
$\bar{C}^{\pm}(\bar{P})C^{\mp}(P)Z_\mu(k)$	$\mp igc_W\left[\left(t_W^2-\frac{2}{\xi}\right)P + \left(1-\frac{2}{\xi}\right)\bar{P}\right]_\mu$
$\bar{C}^A(\bar{P})C^{\pm}(P)W_\mu^{\mp}(k)$	$\mp igs_W\left[\left(1-\frac{2}{\xi}\right)\bar{P} - \frac{2}{\xi}P\right]_\mu$
$\bar{C}^{\pm}(\bar{P})C^A(P)W_\mu^{\mp}(k)$	$\pm igs_W\left(1-\frac{2}{\xi}\right)\left(\bar{P}+P\right)_\mu$
$\bar{C}^Z(\bar{P})C^{\pm}(P)W_\mu^{\mp}(k)$	$\mp igc_W\left[\left(1-\frac{2}{\xi}\right)\bar{P} - \frac{2}{\xi}P\right]_\mu$
$\bar{C}^{\pm}(\bar{P})C^Z(P)W_\mu^{\mp}(k)$	$\mp igc_W\left[\left(t_W^2+\frac{2}{\xi}\right)P - \left(1-\frac{2}{\xi}\right)\bar{P}\right]_\mu$

TABLE IV: Couplings involving four ghosts. This class of couplings are arise as a consequence of using the BRST formalism.

Coupling	Vertex Function	Coupling	Vertex Function
$\bar{C}^+\bar{C}^-C^+C^-$	$2ig^2(1-\frac{2}{\xi})$		
$\bar{C}^Z\bar{C}^{\pm}C^{\mp}C^A$	$2ig^2(1-\frac{2}{\xi})s_Wc_W$	$\bar{C}^A\bar{C}^{\pm}C^{\mp}C^Z$	$2ig^2(1-\frac{2}{\xi})s_Wc_W$
$\bar{C}^A\bar{C}^{\pm}C^{\mp}C^A$	$2ig^2(1-\frac{2}{\xi})s_W^2$	$\bar{C}^Z\bar{C}^{\pm}C^{\mp}C^Z$	$2ig^2(1-\frac{2}{\xi})c_W^2$

We can see that the complete ghost sector is invariant under the $U_e(1)$ group. This gives rise to some vertices which are not present in a linear gauge. For instance, the vertex $\bar{C}^{\pm}C^{\mp}\gamma\gamma$ is a direct consequence of $U_e(1)$-gauge invariance. It is clear thus that the ghost fields satisfy QED-like Ward identities, which can simplify considerably some loop calculations. Since this sector is strongly affected by the nonlinear gauge–fixing procedure, we present all the Feynman rules. They are given in Tables III, IV, V, and VI. In these Tables, all the momenta are token incoming.

IV. SUMMARY

In this paper, we have presented a nonlinear R_ξ gauge for the two Higgs doublet model based in the BRST symmetry and covariance under the electromagnetic gauge group. This gauge–fixing procedure allows one to remove of the theory the unphysical vertices $WG_W\gamma$, WG_WZ, $HWG_W\gamma$, $hWG_W\gamma$, $G_ZWG_W\gamma$, HWG_WZ, hWG_WZ, and G_ZWG_WZ. Due to the covariance of the gauge–fixing procedure under the electromagnetic gauge group, all the charged particles

TABLE V: Couplings generated by two ghosts and two scalar fields. This class of couplings are absent in conventional linear gauges.

Coupling	Vertex Function	Coupling	Vertex Function
$\bar{C}^{\pm}C^AG_W^{\mp}H$	$\frac{ig^2}{2}(\xi-2)s_Wc_{\beta-\alpha}$	$\bar{C}^AC^{\pm}G_W^{\mp}H$	$-ig^2s_Wc_{\beta-\alpha}$
$\bar{C}^{\pm}C^AG_W^{\mp}h$	$\frac{ig^2}{2}(\xi-2)s_Ws_{\beta-\alpha}$	$\bar{C}^AC^{\pm}G_W^{\mp}h$	$-ig^2s_Ws_{\beta-\alpha}$
$\bar{C}^{\pm}C^AG_W^{\mp}G_Z$	$\mp\frac{g^2}{2}(\xi-2)s_W$	$\bar{C}^AC^{\pm}G_W^{\mp}G_Z$	$\pm g^2s_W$
$\bar{C}^{\pm}C^ZG_W^{\mp}H$	$\frac{ig^2}{2}(\xi-2)c_Wc_{\beta-\alpha}$	$\bar{C}^ZC^{\pm}G_W^{\mp}H$	$-ig^2c_Wc_{\beta-\alpha}$
$\bar{C}^{\pm}C^ZG_W^{\mp}h$	$\frac{ig^2}{2}(\xi-2)c_Ws_{\beta-\alpha}$	$\bar{C}^ZC^{\pm}G_W^{\mp}h$	$-ig^2c_Ws_{\beta-\alpha}$
$\bar{C}^{\pm}C^ZG_W^{\mp}G_Z$	$\mp\frac{g^2}{2}(\xi-2)c_W$	$\bar{C}^ZC^{\pm}G_W^{\mp}G_Z$	$\pm g^2c_W$
$\bar{C}^{\pm}C^{\mp}HH$	$\frac{ig^2\xi}{2}c_{\beta-\alpha}^2$	$\bar{C}^{\pm}C^{\mp}hh$	$\frac{ig^2\xi}{2}s_{\beta-\alpha}^2$
$\bar{C}^{\pm}C^{\mp}Hh$	$\frac{ig^2\xi}{2}c_{\beta-\alpha}s_{\beta-\alpha}$	$\bar{C}^{\pm}C^{\mp}G_ZG_Z$	$\frac{ig^2\xi}{2}$
$\bar{C}^{\pm}C^{\mp}G_W^{\pm}G_W^{\mp}$	$-\frac{ig^2\xi}{2}$		

TABLE VI: Quartic couplings generated by two ghosts and two vector bosons. This class of couplings are absent in conventional linear gauges.

Coupling	Vertex Function	Coupling	Vertex Function
$\bar{C}^{\pm}C^{\mp}Z_{\mu}A_{\nu}$	$ig^2 t_W c_{2W} g_{\mu\nu}$		
$\bar{C}^{\pm}C^{\mp}A_{\mu}A_{\nu}$	$2ig^2 s_W^2 g_{\mu\nu}$	$\bar{C}^{\pm}C^{\mp}Z_{\mu}Z_{\nu}$	$-2ig^2 s_W^2 g_{\mu\nu}$
$\bar{C}^{A}C^{\pm}A_{\mu}W_{\nu}^{\mp}$	$\frac{-2i}{\xi}g^2 s_W^2 g_{\mu\nu}$	$\bar{C}^{\pm}C^{A}A_{\mu}W_{\nu}^{\mp}$	$\frac{-i(\xi-2)}{\xi}g^2 s_W^2 g_{\mu\nu}$
$\bar{C}^{A}C^{\pm}Z_{\mu}W_{\nu}^{\mp}$	$\frac{2i}{\xi}g^2 s_W^2 t_W g_{\mu\nu}$	$\bar{C}^{\pm}C^{A}Z_{\mu}W_{\nu}^{\mp}$	$\frac{i(\xi-2)}{\xi}g^2 s_W^2 t_W g_{\mu\nu}$
$\bar{C}^{Z}C^{\pm}Z_{\mu}W_{\nu}^{\mp}$	$\frac{2i}{\xi}g^2 s_W^2 g_{\mu\nu}$	$\bar{C}^{\pm}C^{Z}Z_{\mu}W_{\nu}^{\mp}$	$\frac{i(\xi-2)}{\xi}g^2 s_W^2 g_{\mu\nu}$
$\bar{C}^{Z}C^{\pm}A_{\mu}W_{\nu}^{\mp}$	$\frac{-2i}{\xi}g^2 s_W c_W g_{\mu\nu}$	$\bar{C}^{\pm}C^{Z}A_{\mu}W_{\nu}^{\mp}$	$\frac{-i(\xi-2)}{\xi}g^2 s_W c_W g_{\mu\nu}$

of the model satisfy simple Ward identities, which greatly simplifies the loop calculations. This type of gauges are particulary useful in calculating loop processes that involves at least an external photon. The main advantages are: (a) the number of Feynman diagrams for a given process reduces considerably with respect to those appearing in conventional linear gauges, (b) each type of charged particle circulating in the loop generates by itself electromagnetic gauge invariance, and (c) for those one–loop processes that are free of ultraviolet divergences, such as interactions among neutral particles and photons, the cancelation of divergences also occurs in a simple way through of subsets of diagrams involving the same type of charged particle. The Feynman rules for the new vertices arising in this gauge, as well as for those already present in conventional linear gauge but that are modified by this gauge–fixing procedure, are presented.

[1] G. t'Hooft and M. J. G. Veltman, Nucl. Phys. **B50**, 318 (1972); K. Fujikawa, B. W. Lee, A. I. Sanda, Phys. Rev. **D6**, 2923 (1972); B. W. Lee and J. Zinn-Justin, Phys. Rev. **D5**, 3121, 3137, 3155 (1972); **D7**, 1049 (1972).
[2] C. Becchi, A. Rouet, and A. Stora, Commun. Math. Phys. **42**, 127 (1975); Ann. Phys. (NY) **98**, 287 (1976); I. V. Tyutin: Preprint FIAN (P. N.: Lebedev Physical Institute of the USSR Academy of Science), No. **39** (1975).
[3] K. Fujikawa, Phys. Rev. **D7**, 393 (1973).
[4] See for instance: U. Cotti, J. L. Díaz–Cruz, and J. J. Toscano, Phys. Rev. **D62**, 035009 (2000); J. Hernández–Sánchez, M. A. Pérez, G. Tavares–Velasco, and J. J. Toscano, Phys. Rev. **D69**, 095008 (2004), and references therein.
[5] J. Montaño, F. Ramírez–Zavaleta, G. Tavares–Velascvo, and J. J. Toscano, Phys. Rev. **D72**, 055023 (2005); F. Ramírez–Zavaleta, G. Tavares–Velascvo, and J. J. Toscano, Phys. Rev. **D75**, 075008 (2007).
[6] F. Pisano and V. Pleitez, Phys. Rev. **D46**, 410 (1992); P. H. Prampton, Phys. Rev. Lett. **69**, 2889 (1992).
[7] G. V. Jikia, Nucl. Phys. **B412**, 57 (1994).
[8] Shou Hua Zhu, Chong Sheng Li, Chong Shou Gao, Phys. Rev. **D58**, 015006-1 (1998); Zhou Ya-Jin, Ma Wen-Gan, Hou Hong-Sheng, Zhang Ren-You, Zhou Pei-Jun, Sun Yan-Bin, Phys. Rev. **D68**, 093004 (2003).
[9] Sun La-Zhen and Liu Yao-Yang, Phys. Rev. **D54**, 3563 (1996); Shou-Hua Zho, J. Phys. **G24**, 1703 (1998).
[10] M. Bace and N. D. Hari Dass, Ann. Phys. (NY) **94**, 349 (1975); M. B. Gavela, G. Girardi, C. Malleville, P. Sorba, Nucl. Phys. **B193**, 257 (1981); N. M. Moyonko, J. H. Reid, A. Sen, Phys. Lett. **B136**, 265 (1984); N. M. Moyonko and J. H. Reid, Phys. Rev. **D32**, 962 (1985); J. M. Hernández, M. A. Pérez, G. Tavares-Velasco, J. J. Toscano, Phys. Rev. **D60**, 013004 (1999); M. Baillargen and F. Boudjema, Phys. Lett. **B317**, 371 (1993); U. Cotti, J. L. Díaz-Cruz, J. J. Toscano, Phys. Lett. **B404**, 308 (1997). See also Refs. [**? ? ?**].
[11] See for instance, U. Cotti, J. L. Díaz-Cruz, J. J. Toscano, Phys. Rev. **D62**, 035009 (2000); J. Hernández-Sánchez, M. A. Pérez, G. Tavares-Velasco, J. J. Toscano, Phys Rev. **D69**, 095008 (2004).
[12] J. L. Díaz-Cruz, J. Hernández-Sánchez, J. J. Toscano, Phys. Lett. **B512**, 339 (2001).
[13] L. D. Faddev and V. N. Popov, Phys. Lett. **B25**, 29 (1967); B. S. De Witt, Phys. Rev. **162**, 1195 (1967); L. D. Faddeev, Theor. Mat. Fiz. **1**, 3 (1969)[Theor. Math. Phys. **1**, 1 (1970)].
[14] H. Min, T. Lee, P. Y. Pac, Phys. Rev. **D32**, 440 (1985); H. Hata and I. Niigata, Nucl Phys. **B389**, 133(1993); K. -I. Kondo, Phys. Rev. **D58**, 105019 (1998); T. Shinohara, T. Imai, K. -I. Kondo, Int. J. Mod. Phys. **A18**, 5733 (2003).
[15] J. G. Méndez and J. J. Toscano, Rev. Mex. de Fís. **50**, 346 (2004).
[16] J. F. Gunion and H. E. Haber, Phys. Rev. **D67**, 075019 (2003).
[17] S. Kanemura and H. -A. Tohyama, Phys. Rev. **D57**, 2949 (1998); S. Kanemura, Phys. Rev. **D61**, 09501 (2000); Eur. Phys. J. **C17**, 473 (2000).
[18] See N. Nakanishi and I. Ojima, *Covariant operator formalism of gauge theories and quantum gravity* (World Scientific, 1990) and references therein.
[19] For a review see, J. Gomis, J. Paris, and S. Samuel, Phys. Rep. **259**, 1-145 (1995) and references therein.
[20] Couture, G. and Ng, John N, Z. Phys. **C32**, 579 (1986).
[21] A. Mndez and A. Pomarol, Phys. Lett. **B279**, 98-105 (1992).

[22] Cheung, Kingman and Kong, Otto C. W. Phys. Rev. **D68**, 053003 (2003).
[23] Arhrib, A. and Moultaka, G. Nucl. Phys. **B558**, 3-40 (1999).
[24] Malinsky, Michal and Horejsi, Jiri. Eur. Phys. J. **C34**, 477-486 (2004).